Liquid Assets in a Cash-in-Advance Model

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July 2014

Abstract

I construct a model where both money and a fraction of real assets can be used to purchase consumption goods, and are therefore considered as liquid. I investigate how this set up of competing media of exchange affects the static allocation of real assets and I document the dynamic response of the endogenous variables to shocks. I find that asset holdings increase when a larger fraction of the asset can be used for transactions, and that the effect increases with inflation. Also, with higher inflation, the liquidity premium of the asset increases. The dynamic response of the real variables remains close to the standard model for most variables, but the nominal interest rate reacts much stronger.

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1 Introduction

The analysis of monetary policy and the interaction between real and nominal variables have long been a subject of study in macroeconomics. To discuss monetary policy, nominal features and liquidity issues, money has to be introduced into general equilibrium macro models. One way to do so is to impose that money is required to purchase consumption goods, a framework known as cash-in-advance models.

The cash-in-advance is a useful framework to model monetary policy at the aggregate level. The model has initially been introduced by Lucas (1982) in order to study the determination of prices, interest rates, and exchange rates. In the present paper, the cash-in-advance constraint serves to model the existence of money in a stripped-down version of the real business cycle model.

The reason to use a cash-in-advance model instead of the now widely adopted New Keynesian framework lies in the fundamental role that money plays in the two frameworks. In the New Keynesian context, money is a unit of account, and arises because of the nominal rigidities in the models, most often because of price and/or wage stickiness. Such a model is of limited use when trying to model the liquidity of assets.

Although there exist numerous extensions of the original model proposed by Lucas (1982), there is no model which allows that at least a fraction of real assets can be used in transactions. This paper aims to answer the question how the static allocation and dynamic response to shocks of the model change once a real asset with some degree of liquidity is being introduced. Throughout the paper I use the term liquidity to describe the ability to purchase consumption goods. In other words, a liquid asset provides liquidity services to its holder. In equilibrium then, an asset will not only be valued according to its fundamental value (the discounted future payoffs). Instead, also the fact that the asset can be used in transactions will be valued, potentially leading to a liquidity premium.

The original version of the CIA model assumes that agents can only use real money balances taken over from the last period to purchase goods in the current period. In the model proposed by Lucas (1982), the bond market opens first and then the goods market opens. So, agents allocate their portfolio between cash and bonds at the beginning of each period, after observing any current shocks but prior to purchasing goods. In this model, households would never bring excessive cash to the next period, and thus the cash-in-advance constraint always binds. Svensson (1985) changed the timing of the model slightly, and he assumes that the goods market opens before the bond market. Therefore, for precautionary reasons, the household brings too much money to the next period, sometimes leading to a slack cash-in-advance constraint. In this paper I adopt the version proposed by Lucas (1982).

Lucas and Stokey (1987) analyse a model where the use of money is motivated by the introduction of a cash-in-advance constraint only for a subset of consumption goods. The distinction of cash and credit goods leads to a time-varying velocity of money. Since the household have to use cash for the cash good, they lose the interest rate. If the interest rate is high, the household tends to consume more of the credit good.

An analysis similar to this paper is led by Lagos and Rocheteau (2008), who consider an environment where money and capital act as competing media of exchanges. Different than in this paper, the authors employ a search-theoretic approach to introduce the potentially beneficial role of money. They establish the conditions under which fiat money can be valued, that is when a monetary equilibrium exists. If the condition is satisfied, then liquid capital is being traded at a liquidity premium in the nonmonetary equilibrium.

However, the discussion about productive assets that are used as means of transactions dates back to earlier research. For example, Sargent and Wallace (1983) study the efficiency of commodity money systems, investigate which assets may naturally emerge as commodity money and how the commodity money competes with (interest-bearing) inside money.

There are also real world examples where productive assets have been used as currency. Einzig (as cited in Lagos and Rocheteau, 2008) finds that "Goats and cattle were until comparatively recently the principal currency of a large part of Kenya." Also metal coins could be considered as assets which were either used in the production process or served as medium of exchange in transactions.

The paper is organised as follows. In section 2 I present the model and discuss the differences compared to the simple cash-in-advance model. In section 3 I let the liquidity parameter to fluctuate over time and discuss the implications of the model.

2 Model

There is one representative household that lives forever. Time is discrete. The household maximizes its lifetime utility

$$U = \max \sum_{t=0}^{\infty} u(c_t) \tag{1}$$

with $u(c) = \log(c)$ where c_t is real consumption. The household owns a technology which produces output y. The production function is $y = f(a)$ and requires as input real asset, a. I will use the production function $f(a) = za^{\kappa}$, with $0 < \kappa < 1$, where z is a measure of productivity and follows the AR(1) process

$$\log z_t = \rho_z \log z_{t-1} + \varepsilon_t^z \tag{2}$$

The stock of real assets depreciates at rate δ and depreciation occurs after production takes place. Furthermore, the household can acquire risk-free nominal bonds B_{t+1} in period t, which yield $(1 + i_{t+1})B_{t+1}$ in the next period. Finally, in period t the household also chooses to bring the amount of money, M_{t+1}, to the next period.

The household's period t budget constraint is then

$$c_t + \frac{M_{t+1}}{P_t} + \frac{B_{t+1}}{P_t} + a_{t+1} = f[(1-\phi)a_t] + (1-\delta)a_t + \frac{M_t}{P_t} + (1+i_t)\frac{B_t}{P_t} + \tau_t \tag{3}$$

where P_t is the price level in period t and τ_t are transfers. The parameter $(1 - \phi)$ will be explained further down.

The cash-in-advance (CIA) constraint differs from the standard CIA model described by Lucas (1982) and Svensson (1985). Here, I implement a further element, not present in the existing forms of CIA models. To model the assumption that the asset also provides liquidity services, I allow a fraction ϕ of the asset being used for purchases of the current period consumption. Then, the cash-in-advance constraint is

$$c_t \leq M_t/P_t + \phi a_t \tag{4}$$

where $0 < \phi < 1$ measures the fraction of the asset that is immediately available for purchases. However, as the budget constraint in Equation (3) highlights, the amount of the asset which is used for purchases is no longer available for productive purposes, hence $f[(1 - \phi)a_t]$. Therefore, there is a trade-off between the productive and the liquidity providing usage of the asset. The reason for this is the timing convention. I assume that in order to purchase goods, the agent has to put down the amount the purchase requires, and only then production takes place. At this point, the asset which is used for the purchase can not be used in the production process.

The Bellman equation for this problem is

$$
\begin{aligned}
V(a_t, M_t, B_t) = u(c_t) &+ \beta E_t V(a_{t+1}, M_{t+1}, B_{t+1}) \\
&+ \lambda_t \left\{ f[(1-\phi)a_t] + (1-\delta)a_t + \frac{M_t}{P_t} + \tau_t + (1+i_t)\frac{B_t}{P_t} - c_t - \frac{M_{t+1}}{P_t} + \frac{B_{t+1}}{P_t} - a_{t+1} \right\} \\
&+ \mu_t \left[c_t - M_t/P_t - \phi a_t \right]
\end{aligned} \tag{5}
$$

The first-order conditions

- c_t

$$\frac{1}{c_t} = \lambda_t + \mu_t \tag{6}$$

- a_{t+1}

$$\lambda_t = \beta V_a(a_{t+1}, m_{t+1}, b_{t+1})$$

- B_{t+1}

$$\frac{\lambda_t}{P_t} = \beta V_b(a_{t+1}, m_{t+1}, b_{t+1})$$

- M_{t+1}

$$\frac{\lambda_t}{P_t} = \beta V_m(a_{t+1}, m_{t+1}, b_{t+1})$$

and the following envelope conditions

$$V_a(a_t, M_t, B_t) = \lambda_t \left[(1-\phi)f'[(1-\phi)a_t] + (1-\delta) \right] + \mu_t \phi$$

$$V_b(a_t, M_t, B_t) = \frac{\lambda_t}{P_t}(1+i_t)$$

$$V_m(a_t, M_t, B_t) = \frac{\lambda_t + \mu_t}{P_t}$$

Combining the FOCs of a, B, and M together with the respective envelope conditions yields

$$\lambda_t = \beta\lambda_{t+1}\left[(1-\phi)f'[(1-\phi)a_t] + (1-\delta)\right] + \beta\mu_{t+1}\phi \tag{A}$$

$$\lambda_t\Pi_{t+1} = \beta\lambda_{t+1}(1+i_{t+1}) \tag{B}$$

$$\lambda_t\Pi_{t+1} = \beta\left[\lambda_{t+1} + \mu_{t+1}\right] \tag{M}$$

where $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$ is the gross inflation rate between periods t and $t+1$.

Equation (A) describes the inter-temporal trade-off between renouncing on one unit of wealth today and investing it into the real asset. Tomorrow, the real asset has a return from the production usage, plus the non-depreciated amount. In addition, the real asset also relieves tomorrow's cash-in-advance constraint, measured by μ_{t+1}.

Similarly, Equation (B) describes the inter-temporal trade-off for the nominal bond. Investing one unit in the nominal bond today yields $(1+i_{t+1})$ units tomorrow. Since the bond is nominal, it is being discounted by the gross inflation rate $1/\Pi_{t+1}$.

Finally, Equation (M) describes the trade-off for money. Although money does not yield any return, it also relieves the cash-in-advance constraint. Again, since it is a nominal asset, it is being discounted by the gross inflation rate.

2.1 Equilibrium

Equilibrium is defined as the set of functions $\langle c(\cdot), a(\cdot), M(\cdot), P(\cdot), i(\cdot), \lambda(\cdot), \mu(\cdot)\rangle$ which satisfy the FOC with respect to c_t (Equation (6)), the intertemporal Equations (A), (B), and (M), the CIA constraint described in Equation (4), the market clearing constraint for the asset:

$$c_t + a_{t+1} = f[(1-\phi)a_t] + (1-\delta)a_t \tag{7}$$

and the description of monetary policy, which is assumed to be passive:

$$\log M_{t+1} - \log M_t = (1-\rho_m)\theta + \rho_m\left(\log M_t - \log M_{t-1}\right) + \varepsilon_t^m \tag{8}$$

with $0 < \rho_m < 1$. The nominal variables are not stationary because money supply M grows at a constant rate. However, dividing all nominal variables through the price level yields stationary values. By adding and subtracting $\log P_{t+1} - \log P_t$ on the left hand side of Equation (8) and $\rho_m (\log P_t - \log P_{t-1})$ the nominal variables can be transformed into real variables.

$$
\begin{aligned}
&(\log M_{t+1} - \log P_{t+1}) - (\log M_t - \log P_t) + (\log P_{t+1} - \log P_t) = \\
&\qquad (1-\rho_m)\theta + \rho_m \left[(\log M_t - \log P_t) - (\log M_{t-1} - \log P_{t-1}) + (\log P_t - \log P_{t-1})\right] + \varepsilon_t^m \\
&\log m_{t+1} - \log m_t + \pi_{t+1} = (1-\rho_m)\theta + \rho\left(\log m_t - \log m_{t-1} + \pi_t\right) + \varepsilon_t^m \qquad (9)
\end{aligned}
$$

where $\pi_{t+1} = \Pi_{t+1} - 1$ is the net inflation rate between periods t and $t+1$.

2.2 Steady State

Since the model can not be solved analytically, it is necessary to linearise the model around the steady state. This section describes how the steady state is derived and some of its properties. Since the model describes a closed economy, outstanding bonds have to be zero in steady state, so $B = 0$. From the law of motion of productivity z, described in Equation (2), the steady state value of z is derived

$$
\begin{aligned}
\log z_t &= \rho_z \log z_{t-1} + \varepsilon_t^z \\
z &= 1 \qquad (10)
\end{aligned}
$$

The money supply described in Equation (9) determines the (net) inflation rate π. From the law of motion of the money supply

$$\log m_{t+1} - \log m_t + \pi_{t+1} = (1-\rho_m)\theta + \rho\left(\log m_t - \log m_{t-1} + \pi_t\right) + \varepsilon_t^m$$

So for $m_t = m_{t-1}$, it has to be that $\pi = \theta$, that is inflation is equal to the growth rate of money supply. Since the (nominal) money supply M_t grows at a constant rate, the newly printed money has to be distributed amongst the household. In particular, the transfers to the household in steady state are

$$\tau = m\theta \qquad (11)$$

The price level P, and therefore the amount of real money balances m will be determined by the cash-in-advance constraint.

In steady state, Equation (A) is

$$\lambda = \beta\lambda \left[(1-\phi) f'[(1-\phi)a] + (1-\delta) \right] + \beta\mu\phi \tag{12}$$

and can be combined with Equation (M) to eliminate the Lagrange multipliers

$$1 = \beta \left[(1-\phi) f'[(1-\phi)a] + (1-\delta) \right] + \phi(\Pi - \beta) \tag{13}$$

As $f'[(1-\phi)a] = \kappa z \left[(1-\phi)a\right]^{\kappa-1}$, the expression for real assets in steady state is

$$a = (1-\phi)^{\frac{\kappa}{1-\kappa}} \left[\frac{\beta\kappa z}{1 - \phi(\Pi - \beta) - \beta(1-\delta)} \right]^{\frac{1}{1-\kappa}} \tag{14}$$

Compared to the standard cash-in-advance constraint model without labour decision, the level of assets now depends on the inflation rate Π. Also, a in steady state depends on the newly introduced parameter ϕ. I will discuss the comparative statics in the following subsection.

From the resource constraint, and knowing a from Equation (14), one can then derive the level of consumption in steady state.

$$c = f[(1-\phi)a] - \delta a \tag{15}$$

Combining the first-order condition with respect to consumption (Equation (6)) together with Equation (M) and using Equation (15) yields the Lagrange multiplier λ.

$$\lambda = \frac{\beta}{\Pi} \frac{1}{c} \tag{16}$$

Then, using again Equation (6),

$$\mu = \lambda - \frac{1}{c} \tag{17}$$

Comparing this to the above expression of λ highlights that the cash-in-advance constraint is binding whenever $\Pi > \beta$, that is, whenever inflation exceeds the discount rate.

2.3 Comparative statics

Equation (A) describes the level of assets in steady state. First, I show that the level of the asset is strictly increasing in inflation, Π.

$$\frac{\mathrm{d}a}{\mathrm{d}\Pi} = (1-\phi)^{-\kappa}\frac{\phi a}{1-\kappa} > 0 \tag{18}$$

Since $a > 0$, $0 < \phi < 1$, and $0 < \kappa < 1$, the expression in Equation 18 is unambiguously positive. The reason is that higher inflation induces a higher opportunity cost of money, as the cash-in-advance constraint requires to cover the consumption expenditures with money holdings from the last period. Since the real asset also provides liquidity services to some extent, the relative utility of the asset compared to money increases.

Equation (14) also allows to investigate how the steady state level of real assets depends on the parameter ϕ. In any equilibrium where the agent's cash-in-advance constraint binds, the computation of the partial derivative $\frac{\mathrm{d}a}{\mathrm{d}\phi}$ indicates that for a wide range of sensible parameter values, the steady state level of real assets positively depends on ϕ.[1] That is, the more liquid the asset is, the higher the amount of assets in steady state.

The fact that assets increase with ϕ alone is not surprising, because the set up of the model imposes that with a higher ϕ, the available amount of assets for productive uses decrease. Therefore, the interesting feature of the model is whether the amount of assets available for production, $(1-\phi)a$, move with a change in ϕ. The derivative is equal to

$$\frac{\mathrm{d}(1-\phi)a}{\mathrm{d}\phi} = \frac{a}{1-\kappa}\left\{-\frac{1}{1-\phi} + \frac{\Pi-\beta}{1-\phi(\Pi-\beta)-\beta(1-\delta)}\right\} \lesseqgtr 0 \tag{19}$$

Whether (19) is positive or negative, depends crucially on the steady state money growth rate θ, and therefore the inflation rate, Π. Keeping all other parameters as given in Table 1, I find that $\frac{\mathrm{d}a}{\mathrm{d}\phi} > 0$ if the money growth rate exceeds the $\underline{\theta} = 0.0238$. For sufficient high steady state inflation then, production increases with ϕ.

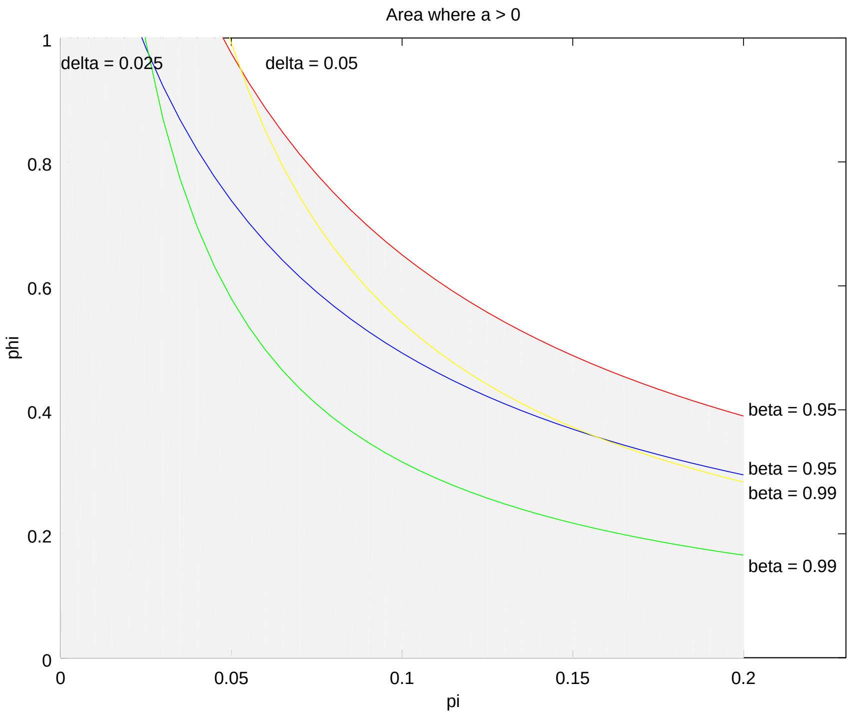
This can be justified by the fact that the asset bears a liquidity premium, in addition to the real return from production. However, whether the benefit from this liquidity services are big enough, depends on the cost that holding money involves. In particular, if inflation is

[1] Details on the parameter range where $\frac{\mathrm{d}a}{\mathrm{d}\phi} > 0$ are reported in the Appendix.

sufficiently low, the inflation tax is low enough and the cost of increasing assets is too high. If instead inflation is sufficiently high, then the asset provides useful liquidity, and the household is willing to accept a lower marginal return on the asset.

To conclude the discussion of Equation (14), note that not all value of the parameters ensure a solution. The relevant part is the numerator in the expression, which has to be positive in order to yield a real solution. Figure 1 documents the area where particular values of the deep parameters β, δ, Π, and ϕ lead to a solution where $a > 0$.

Figure 1: Area where $a > 0$



2.4 Calibration

The parameters are calibrated so that one period reflects a quarter. Where feasible, the values of the deep parameters are taken from Cooley and Hansen (1989) and are standard in the literature[2]. The particular choice of the log utility function already restricts the coefficient of risk aversion to be $\sigma = 1$. The discount factor β, the exponent in the production function κ, and the depreciation rate δ are the same as used by Cooley and Hansen (1989).

The autoregressive coefficients on the technology and the money supply shock as well as the standard deviations of these shocks are taken over from Cooley and Hansen (1989).

[2]For example, King and Rebelo (1999) use a very similar specification.

The new parameter introduced in this model is ϕ, and the calibration of this parameter is somewhat arbitrary. Ideally, one would estimate the parameter. However, in the absence of data on what fraction of assets are used to provide liquidity services, I will use alternative values $\phi = 0.05$ and $\phi = 0.1$ below.

Table 1: Calibration baseline model

Variable	Description	Value
β	Discount factor	0.99
δ	Depreciation rate	0.025
κ	Production function	0.33
θ	Growth rate of money supply	0.03
ϕ	Liquidity parameter of asset	$\{0.05, 0.1\}$
ρ_z	AR(1) of Total Factor Productivity	0.95
ρ_m	AR(1) of money supply	0.48
σ_z	Std. deviation of technology shock	0.00721
σ_m	Std. deviation of money supply shock	0.008

2.5 Dynamics

I log-linearize the model around the steady state described above and solve the model using Dynare. In the following I will compare the impulse response functions to a positive technology and a positive monetary shock of the models for the specifications where $\phi = 0$ (blue, solid line), $\phi = .05$ (red, dashed line), and $\phi = 0.1$ (green, dash-dotted line).

In each case, the linearised system starts at the steady state and is then subject to a one standard deviation technology (ε_t^z) or a monetary (ε_t^m) shock. The figures then show the evolution of the endogenous variables over time as deviations from the steady state.

2.5.1 Productivity shock

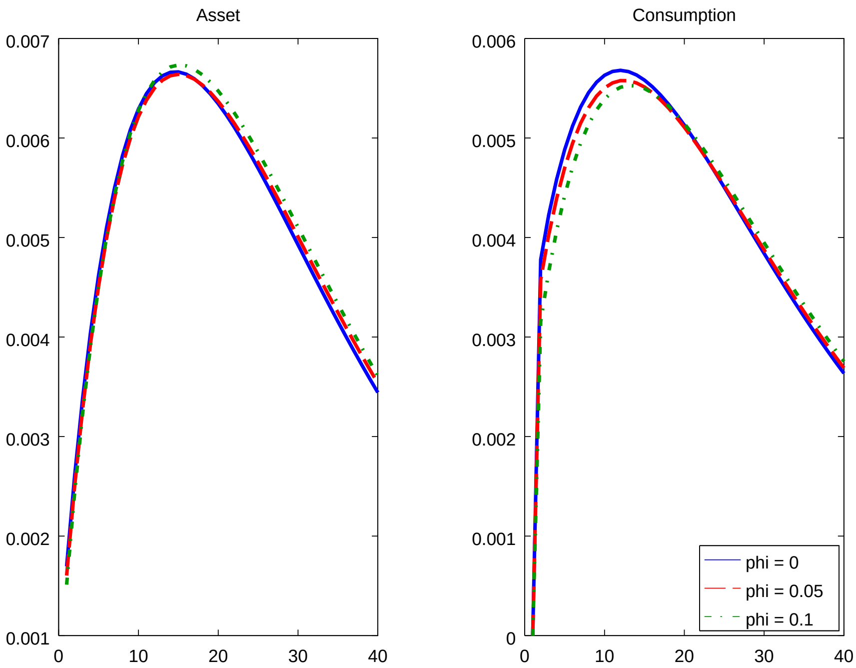
The productivity shock affects the production function through total factor productivity.

$$y = f[(1-\phi)a] = z\,[(1-\phi)a]^{\kappa}$$

A positive productivity shock increases the amount of output for a given input.

Figure 2 shows the impulse response functions of the real variables (asset holdings and consumption) after a technology shock. Since the positive technology shock increases the marginal return of the asset, there is an immediate positive effect on asset holdings. With increasing

Figure 2: IRFs of real variables to technology shock

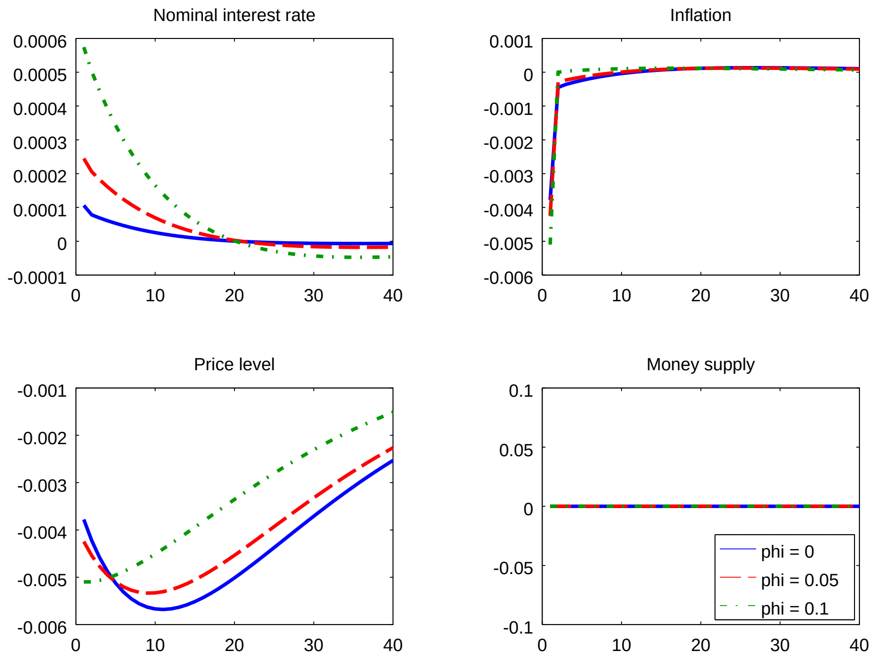


Notes: Impulse response functions to a positive one standard deviation technology shock. Both assets and consumption are shown as percentage deviations from their steady state values ($0.01 = 1\%$).

production, also consumption can be increased by roughly the same percentage amount as the asset. The differences between the specifications of the model ($\phi = \{0, 0.05, 0.1\}$ are rather small. With a higher ϕ, the expansion of asset holdings increases. This is driven by the fact that the asset also carries the liquidity premium.

The response of the nominal variables to a technology shock are shown in Figure 3. In all three specifications, inflation drops immediately after the shock, but recovers quickly to the steady state level. Instead, the price level is more persistent. In all three specifications, the price level initially falls by about 0.5 percent. The initial drop is higher for the specification with $\phi = 0.1$. However, while the impulse response function of the price level for the models with $\phi = 0$ and $\phi = 0.05$, this cannot be seen in the case of $\phi = 0.1$. The differences between the model specifications are mostly visible from the nominal interest. While in the case of $\phi = 0$, the nominal interest rate hardly moves (initially less than 0.01 percentage points, the increase is nearly sixfold for the specification with $\phi = 0.1$.

Figure 3: IRFs of nominal variables to technology shock



Notes: Impulse response functions to a positive one standard deviation technology shock. The nominal interest rate and inflation are shown as absolute deviations from their steady state values (0.01 = 0.01 percentage points). The price level and the money supply are the percentage deviations from their steady state values (0.01 = 1%.

2.5.2 Monetary shock

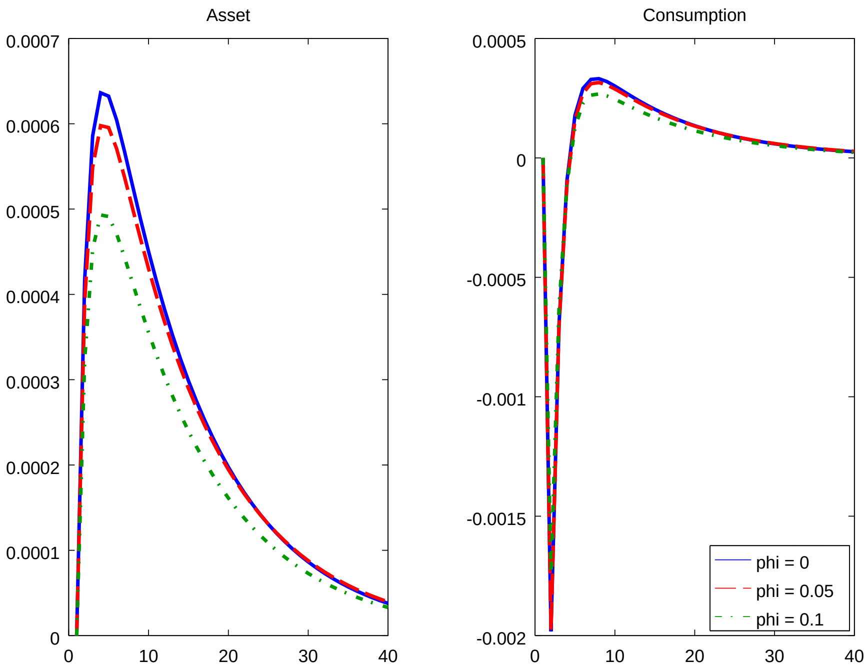
Monetary policy in this model is described by Equation (8),

$$\log M_{t+1} - \log M_t = (1 - \rho_m)\theta + \rho_m (\log M_t - \log M_{t-1}) + \varepsilon_t^m$$

A positive monetary shock ε_t^m temporarily increases the growth rate of (nominal) money supply.

The initial aim of introducing a cash-in-advance constraint was to analyse the linkages between nominal and real variables and to study the effects of monetary policy. Here, monetary policy is simply described as the money supply growth rule and the shock to money supply growth. Figure 4 depicts the impulse response function of the real variables after a positive monetary shock. The left panel shows how the asset increases in response to the monetary shock, while the right panel indicates that consumption initially drops. The reason is that inflation is a tax on holders of money. In the absence of the cash-in-advance constraint, household's would not want to hold money. If the cost of holding money increases because of higher inflation, the household will try to substitute away from money. Due to the constraint they can not

Figure 4: IRFs of real variables to money supply shock

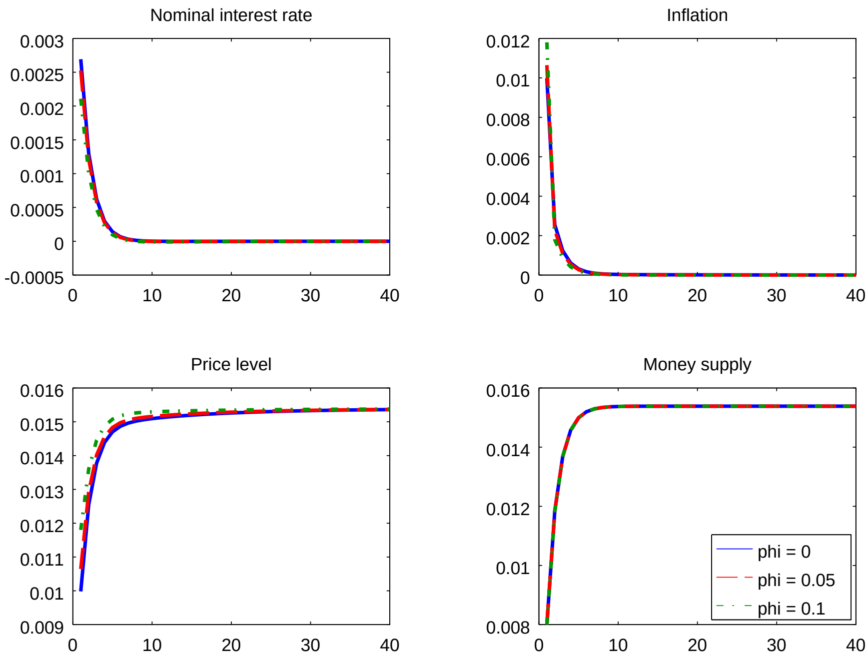


Notes: Impulse response functions to a positive one standard deviation money supply shock. Both assets and consumption are shown as percentage deviations from their steady state values (0.01 = 1%).

substitute money for consumption, so the household will basically substitute money for investment. Therefore, consumption drops in the short-term, while assets increase. The differences between the three specifications are negligible for consumption, but more evident for the asset. For higher ϕ, the asset increases less.

Finally, Figure 5 shows the impulse response functions of the nominal variables to a monetary shock. Money supply M increases briefly as a result of the shock, which in turn increases inflation. Since the AR(1) coefficient in the law of motion of money supply, takes on a value of $\rho_m = 0.5$, the effect of the shock dies out rather quickly. In particular, money supply and inflation are back to their steady state values after just 10 periods. The relative abundance of money drives down the price of the consumption good in monetary terms, therefore leading to a brief drop in the price level. By the Fisher equation, the nominal interest rate increases, as inflation increases and the real interest rate drops slightly (since the asset holdings increase, as documented above). This is consistent with the fact that nominal assets, such as the risk-free bonds, in times of high inflation must be remunerated with a higher nominal interest rate in order to establish an equilibrium.

Figure 5: IRFs of nominal variables to money supply shock



Notes: Impulse response functions to a positive one standard deviation money supply shock. The nominal interest rate and inflation are shown as absolute deviations from their steady state values (0.01 = 0.01 percentage points). The price level and the money supply are the percentage deviations from their steady state values (0.01 = 1%.

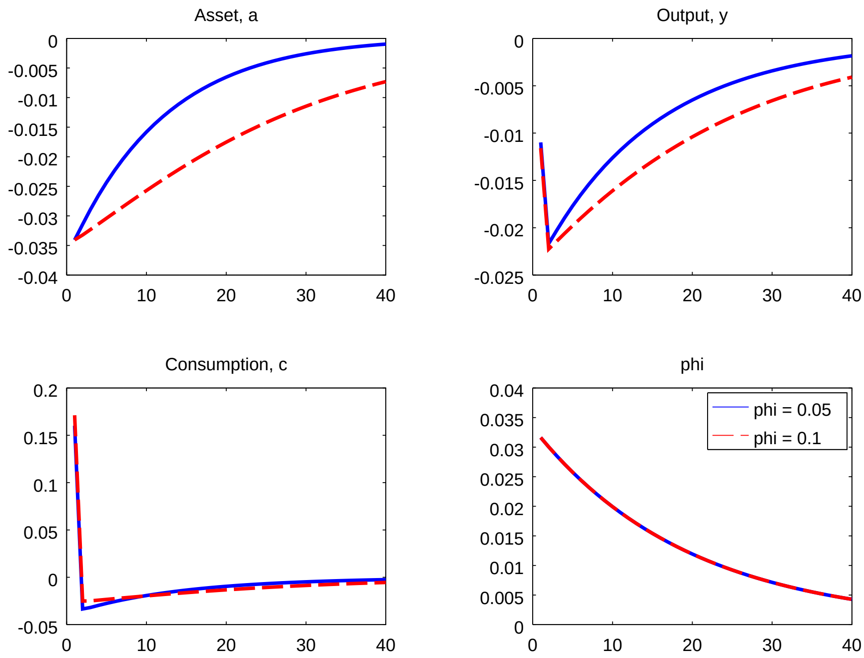
Overall, the introduction of the liquid asset in the cash-in-advance model brings some interesting results. The differences are most notable for the nominal variables in response to a technology shock and for the real variables in response to a monetary shock. Other authors such as King and Watson (1996) have already pointed out that monetary shifts on output are small in cash-in-advance models. This also applies to the present model, especially for the specifications with positive ϕ. The reason for the quasi-neutrality of money in the $\phi > 0$ case is that the liquidity providing feature of the real asset makes the real asset respond even less to monetary shocks.

3 A shock to the liquidity of the real asset

Up to now it was assumed that the parameter ϕ is constant and exogenous. In this section I extend the above model and allow ϕ to vary over time. In particular, I assume that ϕ follows the process

$$\phi_t = (1 - \rho_\phi)\overline{\phi} + \rho_\phi \phi_{t-1} + \varepsilon_t^\phi$$

Figure 6: IRFs of real variables to ϕ-shock

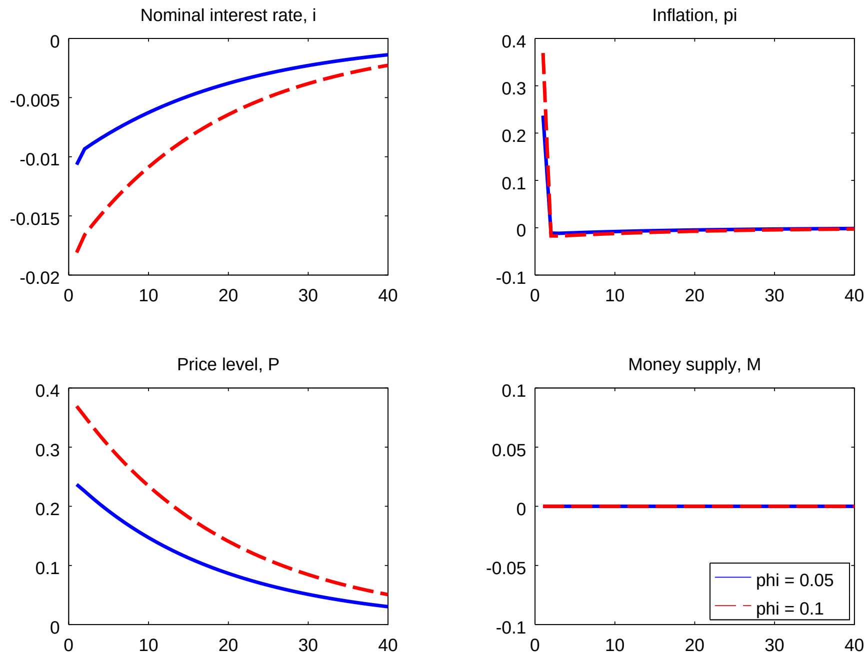


Notes: Impulse response functions to a positive 0.01 unit shock to ϕ. Assets, output and consumption are shown as percentage deviations from their steady state values (0.01 = 1%). ϕ is shown as absolute deviation from its steady state value.

The positive shock to ϕ leads to a significant drop in asset holdings. At first, this seems counterintuitive, since the static analysis of the steady state value of assets in Equation (14) indicated that a depended positively on ϕ. However, Figure 6 indicates the dynamic response to a *temporary* and *unanticipated* shock to ϕ. In particular, although the higher ϕ relieves the cash-in-advance constraint, the immediate effect is that less assets are available for productive uses, which depresses output and therefore the available resources. The household reduces investment into the asset starkly, allowing it to expand consumption for a short period of time. However, the drop in investment must lead to a drop in the asset.

Figure 7 shows the impulse responses of the nominal variables to a shock to ϕ. Notably, the price level and therefore inflation spike on impact.

Figure 7: IRFs of nominal variables to ϕ-shock



Notes: Impulse response functions to a positive 0.01 unit shock to ϕ. The nominal interest rate and inflation are shown as absolute deviations from their steady state values (0.01 = 0.01 percentage points). The price level and the money supply are the percentage deviations from their steady state values (0.01 = 1%).

4 Conclusion

I presented a model which extends the standard cash-in-advance model originally proposed by Lucas (1982). In the standard model, the cash-in-advance constraint requires the representative household to hold money in order to purchase consumption goods. In the model presented here, money is not the only asset which can be used to purchase consumption goods. In particular, I allow that a certain fraction of the real asset can also be employed in such transactions. However, the fraction of the assets used in transactions is not available for consumption in the same period.

The proposed model offers additional features compared to the standard cash-in-advance model. In particular, the real asset is not only valued for its productive usage in the production sector, but there is a second component linked to the liquidity providing feature of the asset. The fact that the real asset can also be used to purchase consumption goods leads even in a risk-less steady state to a positive liquidity premium of the asset, if the inflation rate is high enough.

I also find that the mere fact of having a partially liquid asset does not affect the dynamic response of real and nominal variables to a productivity or a monetary shock. In particular, it remains true that money remains nearly neutral, even with the introduction of the partial liquidity of the real asset. The only exception is the fact that the nominal interest rate reacts much stronger to the shocks. One important finding is that a shock to the fraction of the asset that can be used to purchase consumption goods does have relatively large effects on the real variables. For future research, it would be interesting to measure empirically how much variability in real variables can be explained by a varying ϕ.

The model used in this paper excluded for simplicity many features now usually built into dynamic stochastic general equilibrium models. It would be interesting to investigate the effects of the liquidity providing feature in a more complete model, including for example a labour-leisure decision. Also, one can think about a model where there are multiple real assets, each with a different degree of liquidity. Furthermore, the monetary policy side of the model could be more active. The question arises then, whether the monetary authority are able to (temporary or permanently) mitigate the effects of changes to the liquidity of the asset by monetary policy (conventional or not).

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Appendix

The expression for assets in steady state, as described in Equation (14).

$$a = (1-\phi)^{\frac{\kappa}{1-\kappa}} \left[\frac{1-\phi(\Pi-\beta)-\beta(1-\delta)}{\beta\kappa z} \right]^{\frac{1}{\kappa-1}} \tag{20}$$

The derivative with respect to ϕ

$$\frac{\mathrm{d}a}{\mathrm{d}\phi} = \frac{\kappa}{1-\kappa}(1-\phi)^{\frac{\kappa}{1-\kappa}-1} \cdot (-1) \Big[\dots \Big]^{\frac{1}{\kappa-1}} + \\ + (1-\phi)^{\frac{\kappa}{1-\kappa}} \left(\frac{1}{\kappa-1} \right) \Big[\dots \Big]^{\frac{1}{\kappa-1}-1} \left(\frac{-(\Pi-\beta)}{\beta\kappa} \right)$$

Extracting the common terms

$$\frac{\mathrm{d}a}{\mathrm{d}\phi} = \frac{a}{1-\kappa} \left\{ -\frac{\kappa}{1-\phi} + \frac{\Pi-\beta}{1-\phi(\Pi-\beta)-\beta(1-\delta)} \right\} \tag{21}$$

Table 2 indicates the range of values a parameter can take, for the derivative of a with respect to ϕ to be positive, while all other parameters take the value indicated in the calibration section (see Table 1).

Table 2: Sensitivity

Variable	$\phi = 0.05$		$\phi = 0.1$	
	low	high	low	high
β		1.0322		1.0323
δ		0.2068		0.2110
κ		1.0896		1.0951
θ	-0.0248		-0.0239	
ϕ		1.0385		1.0385